On doing a math[s] project with “Aristotle”

For the SFLean Meetup, July 6, 2026

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Notes on the example project

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Notes on the example project

The accompanying files are part of a repository created by Aristotle (aristotle.harmonic.fun) over a period of less than 3 days in July 2026. They are included without change, and to anthropomorphize, "contain Aristotle's view of the mathematics involved." The wording is generated by Aristotle, and apologies are offered for any inadvertent plagiarism.

The pattern of proof was suggested by the director (Gerhard Paseman) and was followed more or less faithfully by Aristotle. Some specific prompts were given to direct the work produced, and are included below. These prompts and pattern were generated by the director after months of independent research and development (largely without any machine assistance).

The director hopes these notes will guide future development in machine-assisted mathematics, science, and teaching.

Prompts

The first prompt sets the stage for the shape of the project

I have bounds between various quantities to verify or refute. I want to have Lean code output to do formal verification/refutation. I want a markup file (Explain.md) generated that explains each step in the verification/refutation process. I want a markup file (Summary.md) that summarizes the main steps and provides informal intuition for the proofs of the results below. I want a slide presentation (Slides.xxx where xxx can be .md (markup), .pdf (Portable Document Format), or .tex (Latex source)) to use to present the results. In Explain.md, if there are Mathlib theorems or internet resources that are used in the verification/refutation/explanation, I need a bibliography section that collects the use of Mathlib lemmas/internet resources . Finally, if there are uses of these theorems in the literature, I need these collected in a markup file (Applications.md). Here are the quantities and the theorems I want verified or refuted. Let k be a natural number. Let h(k) = ceiling(k/2) be a natural number. Let A(k) = \product_{p prime, h(k)<p<=k} p . Let B(k) = \product_{p prime, there is natural number e with h(k)< p^e <=k }. Let C(k) = {k choose h(k)} = binom(k, h(k)), the binomial coefficient of k over h(k). Let D(k) = 4^{k- h(k)}. Let E(k) = {2*h(k) choose h(k)}. For each pair of A(k), B(k), C(k), D(k), E(k), determine for all k which is larger and which is smaller, or if they are incomparable. Let j_0 = k and j_i satisfy j_{i+1} = h(j_i) for 0<=i<r. When j_r >1, let AA(k,r) = \prod_{0<=i<r} A(j_i), and define BB(k,r), CC(k,r), DD(k,r) and EE(k,r) similarly as products of the B(j_i), C(j_i), D(j_i), and E(j_i). Determine for which k, r is the greater of AA(k,r) and BB(k,r), and of the other pairs taken from {AA(k,r),BB(k,r),CC(k,r),DD(k,r),EE(k,r)}. Many of the results should be phrased using a template like "for all k and r, BB(k,r) >= EE(k,r) except for the following list of pairs (k,r) ... ".

Prompts

The next prompt asks for further work using the same shape:

Thank you. I want to do something similar with the next set of quantities/bounds. I want an Explain file (ExplainPCB.md) a summary file (SummaryPCB.md), a slide presentation (SlidesPCB.xxx) a list of resources (Mathlib and literature) used in ExplainPCB, and if these are used , an Applications file (ApplicationsPCB.md). These quantities relate to PCB, the Bounds on the Prime Counting function \pi(k) = cardinality of({p : p is natural and p is a prime and p<=k}) for natural numbers k. Let a(k) = ceiling(k/2), and let b(k)= 1+ceiling(x/3). Define subsets of natural numbers UNO ={1} , DOS = {(2*n) : n>1}, and TRES = {3*(2n+1) : n>0}. Show that \pi(k)<= a(k) and \pi(k) <= b(k) for all natural numbers k>=0 in two ways: Way I uses the fact that UNO, DOS, and TRES are subsets of the complement set to PRIMES in the naturals, meaning UNO union DOS union TRES has no primes. Also, UNO, DOS and TRES are distinct. Determine the counting functions for UNO union DOS and for UNO union DOS union TRES. Use this counting function to show the counting function of the complement set in the naturals is a(k) respectively b(k). Conclude that pi(k) <= a(k) and pi(k)<= b(k), since the primes are a subset of each complement. Way II: show that \pi(k+2)<= \pi(k)+1 for all k>=2 using parity and conclude that pi(k)<=a(k). Then show that for k>0 pi(3k+3)<= 1 +pi(3k) using parity and the proof of the fact that pi(3*(k+1))= pi(3*k + 2). We will use the Lean code later, so for now keep the Lean files for the prime count bounds (PCB) separate from the previous files as much as possible.

Prompts

The next prompt continues asking for foundational development:

Thank you. I now want to do the same thing for the following set of results (LOA for Laws Of Ademption). I want similar files (ExplainLOA, SummaryLOA, SlidesLOA, ApplicationsLOA) for these results below. These laws are consequences of a basic result, and the proof structure should use the basic result in the proofs of the remaining results. The results all concern arithmetic progressions with initial term n+d and common difference d, and with k many terms, where n,k,d are natural numbers, d>=1, and gcd(n,d)=1. The d=1 case means the progression n+d,n+2d,n+3d,...n+kd is a set of k consecutive positive natural numbers n+1, n+2,n+3,...,n+k. The basic result is as follows: if p is a prime and p does not divide d, then the power of p dividing \product_{1<=j<=k} (n +j*d) is at most the power of p dividing (k-1)! (say this is p^f where p^f divides (k-1)! and p^(f+1) does not divide (k-1)!)) times the maximal power of p dividing any single term (n+j*d) (say this is p^e where p^e divides n+i*d for some 1<=i<=k and p^(e+1) divides no term (n+j*d) for any 1<=j<=d), so if p^g divides \product_{1<=j<=k} (n +j*d) then p^g divides p^(e+f). if p divides d, then p does not divide n+j*d for any j. Another way to state the basic result: if p divides \product_{1<=j<=k} (n+j*d) where gcd(n,d)=1, and there is i with 1<=i<=k and e such that p^e divides n+i*d and p^(e+1) divides no term n+j*d for 1<=j<=k, and if p^f precisely divides (k-1)!, then if also p^g divides \product_{1<=j<=k} (n+j*d) then p^g divides [(k-1)!]_p * (n+i*d), where [(k-1)!]_p = p^f is that power of p drecisely dividing (k-1)! and (n+i*d) is a term among n+j*d for 1<=j<=k with the highest power of p dividing it (p^e). This basic result has the following consequences: (LOA Erdos) if p^e divides {(n+k) choose k} for natural numbers n and k, then p^e <= n+k; (ELOA for d=1) : if r<k and p_1,...,p_r are distinct primes dividing \product_{1<=j<=k} (n+j) then if also the powers (p_i)^(e_i) divide the product \product_{1<=i<=k} (n+j), then \product_{1<=i<=r} (p_i)^(e_i)<=k! \product_{1<=i<=r} (n+k+1-i); (ELOA for LCM when d>1) : if r<k and p_1,...,p_r are distinct primes dividing \product_{1<=i<=k}(n+j*d) and if also (p_i)^(e_i) each divide the product

\product_{1<=i<=k}(n+j), then LCM_{1<=i<=r} (p_i)^(e_i) <= LCM_{1<=i<=r} [(k-1)!]_(p_i) * LCM_{1<=i<=r} (n + j_i*d), where for each i with 1<=i<=r, j_i is an integer with 1<=j_i<=k such that (n+j_i*d) has the largest prime power factor of p_i among all k terms (n+j*d); (ELOA Principal Lemma) if p_1,..., p_r are distinct primes for r<k all dividing \product_{1<=j<=k} (n+j*d), and if \product_{1<=i<=r} (p_i)^(e_i) divides \product_{1<=j<=k} (n+j*d), then \product_{1<=i<=r} (p_i)^(e_i) <= (k-1)! \product_{1<=i<=r} (n+(k+1-i)*d). Note that in the ELOA versions, the exponents e_i are allowed to be zero, so that it is possible to apply this lemma to primes q which don't divide \product_{1<=j<=k} (n +j*d), as long as in the ELOA the exponent e for q is zero, so that q^e=1.

These prompts were the directions that Aristotle followed to produce the files Explain.md, Applications.md, Summary.md, and Slides.tex, and other files. This project is continuing as part of the Sylvester-Schur Suite, an attempt to auto-formalize and AI-generate material to explain a new proof of results of J.J. Sylvester and I. Schur, motivated by the proofs of these two mathematicians and many following, most notably Erdős Pál . This project also aims at extending and strengthening the results and finding new applications for them

Further progress on the Sylvester-Schur Suite will be reported at later meetings of SF Lean— be sure to check back.

Gerhard Paseman, 2026.07.07.